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# BOX-JENKINS MODELLING OF GLOBAL TEMPERATURES

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## Abstract

It is commonly thought that the onset of the warming trend in global temperatures began around 1970, the year that George Box and Gwilym Jenkins published their famous book "Time Series Analysis: Forecasting and Control". The aim of this paper is to examine what implications relating to current debates concerning global warming can be drawn from an analysis of global temperatures using the techniques developed and proposed by Box and Jenkins. The results of the exercise serve to reinforce the view that conflicting evidence bedevils the time series analysis of the observed temperature record, thus adding further uncertainty to the empirical, if not the scientific, basis for anthropogenic global warming

## 1. Introduction

It is commonly thought that the onset of the warming trend in global temperatures began around 1970, the year that George Box and Gwilym Jenkins published their famous book "*Time Series Analysis*: *Forecasting and Control*" (Box and Jenkins [1]) that 'after 40 years many still regard

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… as the bible of time series analysis' (Mills et al. [10], page 1). This is demonstrated in Figure 1, where the HADCRUT3 annual global temperature record from 1850 to 2010 is plotted with a low-pass trend filter superimposed.[1]

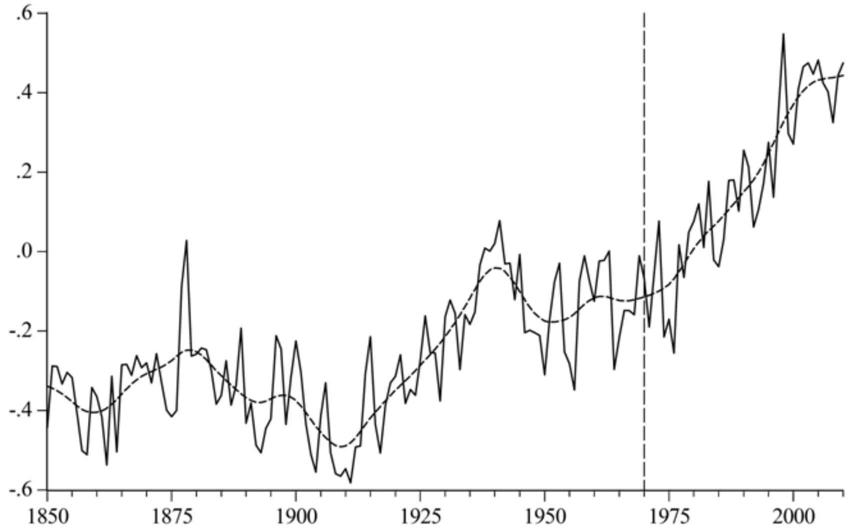


**Figure 1.** HADCRUT3 series: 1850-2010; temperature anomaly (ºC) relative to 1951-1980 mean with low-pass trend filter superimposed.

What implications relating to current debates concerning global warming can be drawn from an analysis of this temperature series by using the techniques proposed by Box and Jenkins, techniques that have since become standard in many areas of applied time series analysis, such as economics, business, and finance, but have only intermittently found their way into the climate science literature (see, for example, Mills [7])?

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[1] See Brohan et al. [2] for details of HADCRUT3's construction: The series was downloaded from www.cru.uea.ac.uk/cru/data/temperature/hadcrut3gl.txt. As the trend filter is employed simply for descriptive purposes, a Hodrick-Prescott [4] filter with smoothing parameter set at 100 is used. This automatically adjusts the filter weights at the ends of the series and so avoids the rather tortuous methods that have been proposed in the climate science literature and for which, there appear to be very little statistical basis: see, for example, Mann [6] and Soon et al. [17].

This is the aim of the present paper, which begins, in Section 2, with the development of an ARIMA model for global temperatures using the familiar three-stage process of identification, estimation, and diagnostic checking. Various forecasting exercises are undertaken in Sections 3 and 4 and these lead to a modification of the underlying drift component of the model to better represent the upward trend in temperatures since 1970. In Section 5, $CO_2$ emissions are introduced and a transfer function linking temperatures to emissions growth is developed. Such a model, which is found to incorporate a rather perverse negative relationship between these variables, may be criticised from a post-Box and Jenkins perspective for concentrating solely on the short-run relationship between temperatures and emissions. Cointegration extensions of the transfer function approach are therefore examined, but these lead to conflicting implications. It is argued in Section 6 that such conflicts are often found to bedevil the analysis of temperature data and serve to add further uncertainty to the empirical basis for anthropogenic global warming.

## 2. Identification, Estimation and Diagnostic Checking of an ARIMA Model for Global Temperatures

In the subsequent analysis, we use, without definition, the terminology and notation introduced by Box and Jenkins [1] that has since become standard in time series analysis. Figure 2 thus shows the SACFs and SPACFs for lags $k \leq 12$ for $z_t$, $\Delta z_t$, and $\Delta^2 z_t$, where $z_t$ is annual temperature. Also shown are two standard error bounds under a null of white noise: with a sample size of $n = 161$, these are given by $2(n-d)^{-1/2}$ and are thus approximately $\pm 0.16$. It is clear that first differencing $(d = 1)$ is indicated, since the SACF for $z_t$ displays a linear decline, indicative of non-stationarity, while the SACF for $\Delta^2 z_t$ has $r_1 \approx -0.5$, which is a clear sign of over-differencing.[2]

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[2] A battery of unit root tests confirms that the series does indeed contain a single unit root: An ADF test including both a linear trend and an intercept, with lag augmentation 3, for example, is just $-2.25$ with an associated $p$-value of 0.46.

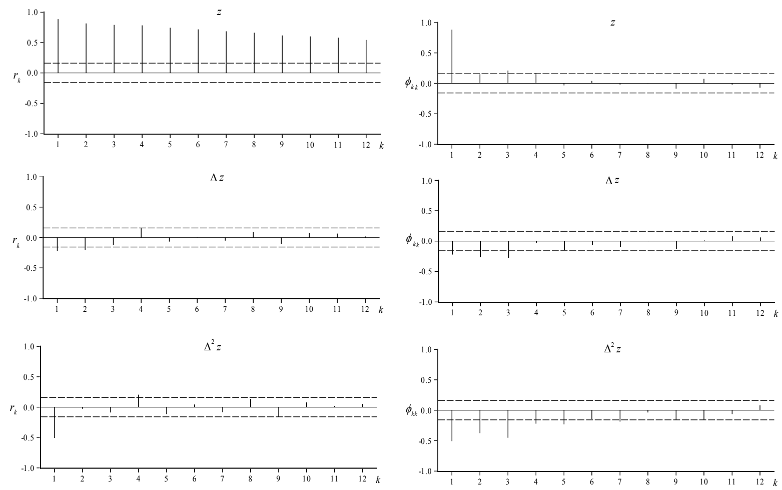


**Figure 2.** SACFs and SPACFs for $d = 0, 1, 2$.

The SACF for $\Delta z_t$ has just $r_1 = -0.22$ and $r_2 = -0.20$ significant, while the SPACF has the significant values $\phi_{11} = -0.22$, $\phi_{22} = -0.26$, and $\phi_{33} = -0.27$. These suggest either an ARIMA (3, 1, 0) or an ARIMA (0, 1, 2) process, although an ARIMA (1, 1, 1) might possibly be entertained. Estimation of these models obtained, respectively,

$$\underset{}{\Delta z_t} = \underset{(\pm 0.0084)}{0.0096} - \underset{(\pm 0.078)}{0.355}\Delta z_{t-1} - \underset{(\pm 0.078)}{0.344}\Delta z_{t-2} - \underset{(\pm 0.078)}{0.274}\Delta z_{t-3} + a_t, \qquad \hat{\sigma} = 0.1047,$$

$$\Delta z_t = \underset{(\pm 0.0027)}{0.0051} + a_t - \underset{(\pm 0.077)}{0.422}a_{t-1} - \underset{(\pm 0.077)}{0.252}a_{t-2}, \qquad \hat{\sigma} = 0.1037,$$

$$\Delta z_t - \underset{(\pm 0.118)}{0.378}\Delta z_{t-1} = \underset{(\pm 0.0017)}{0.0032} + a_t - \underset{(\pm 0.073)}{0.811}a_{t-1}, \qquad \hat{\sigma} = 0.1041,$$

where standard errors are shown in parentheses. The residuals from all three models show no evidence of non-normality and other diagnostic checks revealed no residual autocorrelation or heteroskedasticity. In terms of goodness of fit, the ARIMA $(0, 1, 2)$ is selected and this also has the advantage that the intercept term is significant, albeit at the rather modest 0.065 level ($t$-ratio 1.86), as compared to the intercept of the ARIMA $(3, 1, 0)$, which is only significant at the 0.25 level ($t$-ratio 1.15). Although this intercept is not precisely determined, it will be retained in subsequent analysis as it represents the (upward) drift, although at a rate of just 0.5ºC per century, in global temperatures over the sample period from a random walk with correlated and normally distributed innovations:

$$z_t = z_{t-1} + e_t = \sum\nolimits_{i=0}^{t} e_{t-i}, \quad e_t = a_t - 0.422a_{t-1} - 0.252a_{t-2},$$

$$a_t \sim N\left(0.0051, 0.1037^2\right).$$

This model thus implies that global temperatures are the accumulation of correlated normal random variables having mean 0.0051 and standard error 0.1037. Such models have been well known to time series analysts since the seminal works of Working [18] and Slutsky [16], with the warming trend since 1970, then being attributable simply to natural variation in the random accumulation.

## 3. The Forecasting Performance of the ARIMA(0, 1, 2) Model of Global Temperatures

As set out in Box and Jenkins ([1], Chapter 5), the ARIMA $(0, 1, 2)$ process

$$\Delta z_t = \theta_0 + a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} = \theta_0 + \left(1 - \theta_1 B - \theta_2 B^2\right) a_t,$$

has $l$-step ahead forecasts from origin $t$ that are given by

$$\hat{z}_t(1) = z_t + \theta_0 - \theta_1 a_t - \theta_2 a_{t-1},$$

$$\hat{z}_t(2) = \hat{z}_t(1) + \theta_0 - \theta_2 a_t,$$

$$\hat{z}_t(l) = \hat{z}_t(l-1) + \theta_0, \qquad l \geq 3.$$

For the fitted model, these forecasts are

$$\hat{z}_t(1) = z_t + 0.0051 - 0.422a_t - 0.252a_{t-1},$$

$$\hat{z}_t(2) = \hat{z}_t(1) + 0.0051 - 0.252a_t,$$

$$\hat{z}_t(l) = \hat{z}_t(l-1) + 0.0051, \qquad l \geq 3.$$

The variance of the $l$-step ahead forecast error $a_t(l) = z_t - \hat{z}_t(l)$ is given (on specializing equation (5.1.16) of Box and Jenkins [1], page 128) by

$$V(l) = \left(1 + (1-\theta_1)^2 + (1-\theta_1-\theta_2)^2(l-2)\right)\sigma^2, \qquad l > 1.$$

For the fitted model, this forecast error variance is

$$V(l) = 0.0108(1.334 + 0.106(l-2)).$$

Figure 3 employs this result to show global temperatures along with 0.025 and 0.975 forecast bounds from origin $t = 1850$, calculated as $\hat{z}_{1850}(l) \pm 1.96V(l)^{1/2}$. Thus, if this process actually did generate global temperatures then, in 1850, a 95% forecast interval for the 2010 temperature would be $(-0.461, 1.270)$, with the actual temperature being 0.475. Of course, such a ‘thought experiment’ is rather artificial and serves merely to illustrate the extent of the randomness inherent in this process generating global temperatures. Perhaps more useful are out-of-sample forecasting exercises. Figure 4 shows the 95% forecast intervals from an origin of 1970 using the following ARIMA $(0, 1, 2)$ model fitted to temperatures up to 1970, which passes all the usual diagnostic checks.

$$\Delta z_t = a_t - \underset{(\pm 0.088)}{0.393}a_{t-1} - \underset{(\pm 0.088)}{0.289}a_{t-2}, \qquad \hat{\sigma} = 0.1011.$$

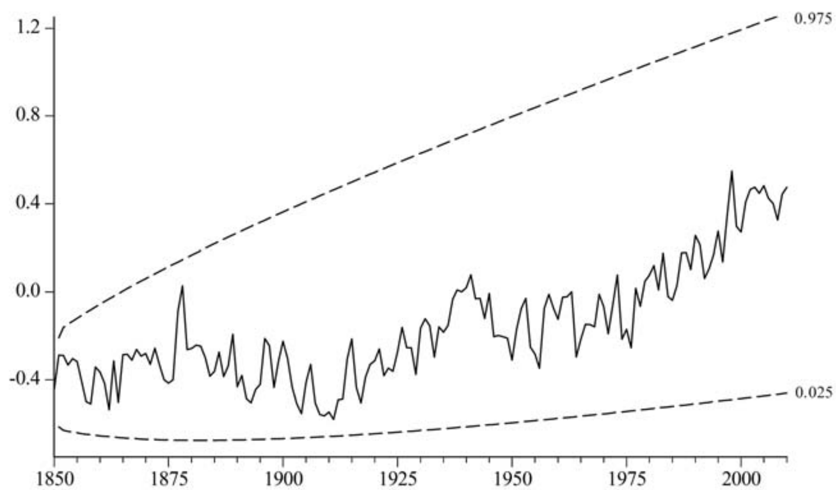


**Figure 3.** Global temperature with 0.025 and 0.975 forecast bounds at origin 1850.

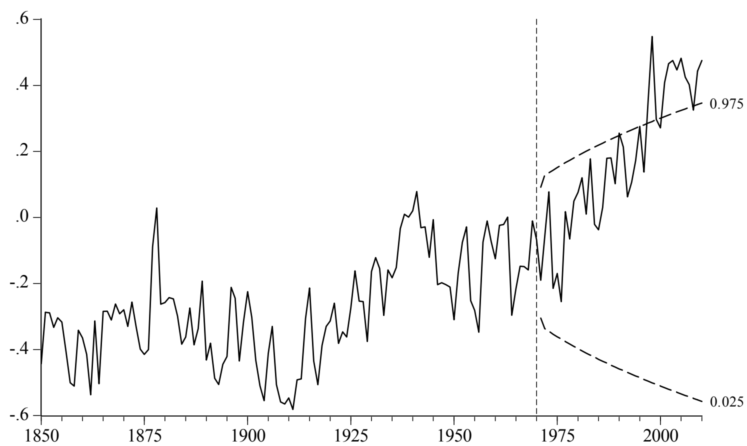


**Figure 4.** Forecasts of global temperature from 1970 origin to 2010 with 0.025 and 0.975 forecast bounds.

Incorporating an intercept produces estimates of $\hat{\theta}_0 = 0.0022(\pm 0.0029)$, $\hat{\theta}_1 = 0.400(\pm 0.088)$, and $\hat{\theta}_2 = 0.296(\pm 0.088)$, so that the model above is clearly appropriate. The actual temperature lies within the 95% forecast interval until 1998, the year of record global temperatures, and since then the interval has been breached on several occasions, although temperatures do seem to have 'plateaued' during the first decade of the 21st century.

## 4. Forecasting 2100 Temperatures

An often quoted conclusion from the Intergovernmental Panel on Climate Change (IPCC) is that global temperatures are projected to increase by between 1.4 and 5.8°C above 1990 levels by 2100 (see, for example, Schneider [15]). The top plot of Figure 5 shows projections from the ARIMA $(0, 1, 2)$, fitted up to 2010, out to 2100, in the form of probability bounds. The point forecast for 2100 is 0.9°C with a central 10% interval spanning 0.85 to 0.95°C expanding out to a 99% interval spanning 0.02 to 1.77°C. Since the 1990 temperature was 0.255°C, a 1.4°C increase takes the projected 2100 temperature to 1.655°C, which, from the intervals calculated in Figure 5, has a probability of just 0.01 of being exceeded.

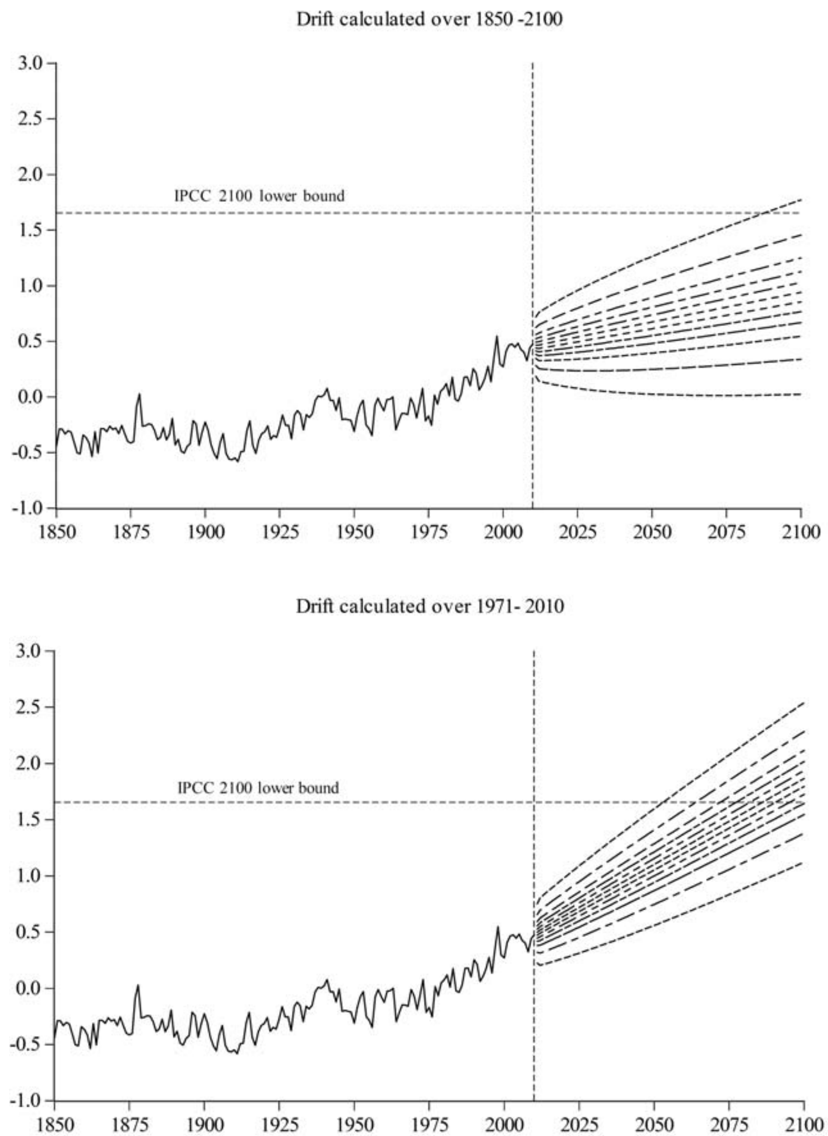


**Figure 5.** Forecast bounds of global temperature from 2010 origin out to 2100 for different drifts. Central bound is 10%, with successive bounds being 30%, 50%, 70%, 95%, and 99%, respectively.

From a comparison of the models fitted up to 1970 and up to 2010, it is clear that the intercept (drift component) has shifted upward to accommodate the warming trend of the last 40 years, with the moving average parameters remaining relatively stable.[3] A shifting trend was thus incorporated by replacing the intercept with an intervention variable, defined as $I_t = 0$ for $t \le 1970$ and $I_t = 1$ for $t > 1970$, leading to

$$\Delta z_t = \underset{(\pm 0.0046)}{0.0151} I_t + a_t - \underset{(\pm 0.077)}{0.459} a_{t-1} - \underset{(\pm 0.077)}{0.281} a_{t-2}, \qquad \hat{\sigma} = 0.1023.$$

The bottom panel of Figure 5 shows the projections from this model, for which the drift is three times larger than before and significant. The IPCC projected 2100 temperature of 1.655ºC now has a 64% probability of being exceeded, with the central forecast for 2100 being 1.83ºC. Even here, however, the chance of temperatures increasing by, say, 2.5ºC over the period 1990 to 2100 is still extremely small, being less than 0.001.

### 5. Incorporating $CO_2$ Emissions into the Model

A central theme in the global warming debate is the influence of fossil-fuel $CO_2$ emissions on temperatures. Figure 6 shows both the levels and logarithms of annual global fossil-fuel $CO_2$ emissions, from which it is clear that taking logarithms approximately linearises the data.[4]

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[3] The break in 1970 is chosen for illustrative purposes only. Clearly, the break date, if it really exists, could itself be estimated, but only at the expense of considerably complicating the analysis, by using the techniques surveyed in Perron [11]. Such an extension is hardly in the spirit of the present paper, however.

[4] The series was downloaded from the Carbon Dioxide Information Analysis Center (CDIAC) website at http://cdiac.ornl.gov/trends/emis/meth_reg.html.

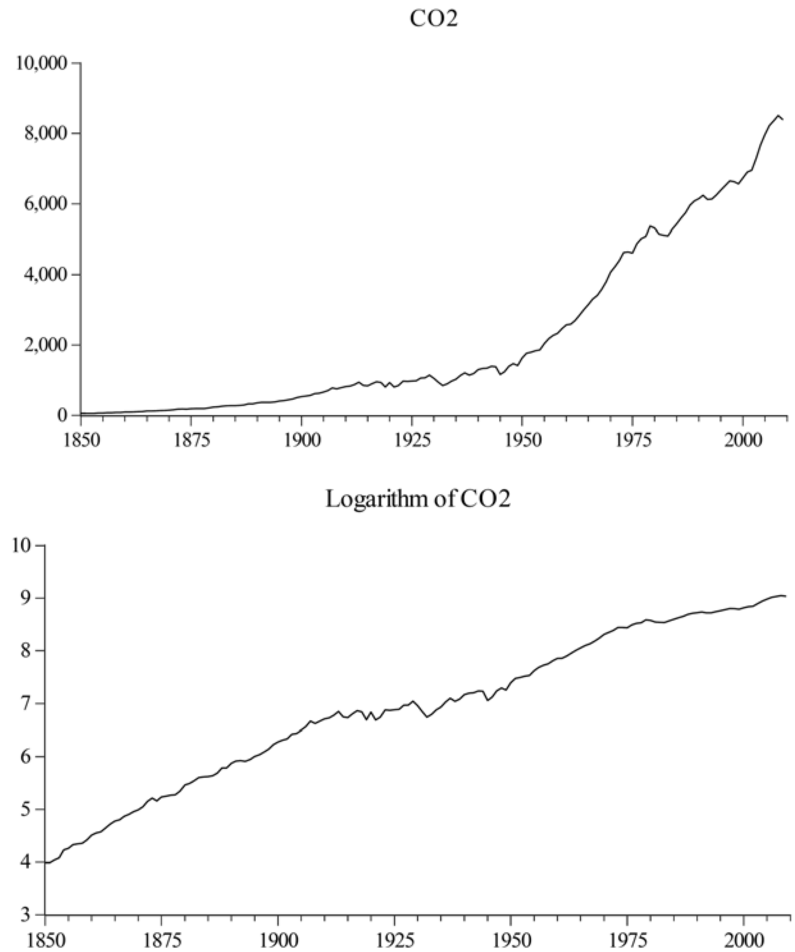


**Figure 6.** Annual fossil-fuel $CO_2$ emissions, 1850-2010.

Denoting the logarithms as $co2_t$, standard Box-Jenkins analysis shows that this series is adequately modelled as a drifting random walk

$$\Delta co2_t = \underset{(\pm 0.0040)}{0.0317} + b_t, \qquad \hat{\sigma}_b = 0.0501,$$

so that emissions grow at an average of 3.2% per annum. Figure 7 shows the cross-correlation function between 'pre-whitened' emissions growth $b_t$

and the residuals from the intervention model, $a_t$, with the correlation between $b_{t-k}$ and $a_t$ being denoted $r_{ab}(k)$, $-12 \le k \le 12$. Under the hypothesis that these are uncorrelated the standard error of $r_{ab}(k)$ is approximately $(n-1)^{-1/2} = 0.08$, so that only $r_{ab}(0)$ and $r_{ab}(4)$ are (just) significant, both being negative.

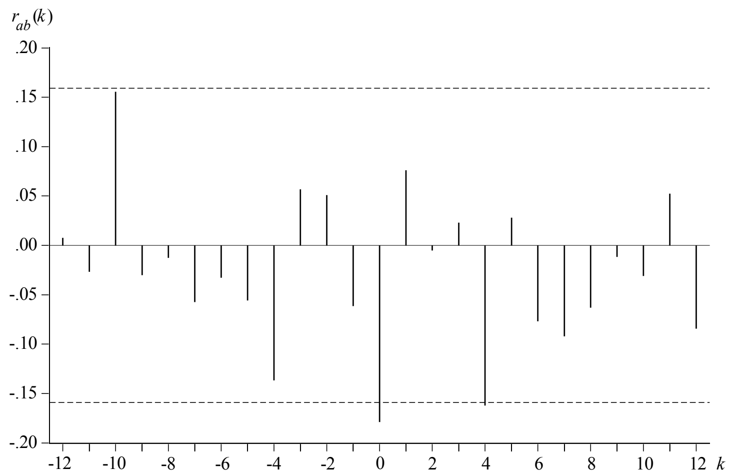


**Figure 7.** Cross-correlation function between prewhitened $z_t$ and $co2_{t-k}$ with two standard error bounds.

After some experimentation, the following transfer function was arrived at using the techniques developed in Box and Jenkins ([1], Chapter 11)

$$\Delta z_t = \underset{(\pm 0.0048)}{0.0148} I_t - \underset{(\pm 0.147)}{0.326} \Delta co2_t + \underset{(\pm 0.146)}{0.327} \Delta co2_{t-1}$$

$$+ a_t - \underset{(\pm 0.078)}{0.442} a_{t-1} - \underset{(\pm 0.078)}{0.284} a_{t-2}, \quad \hat{\sigma} = 0.1016.$$

Inclusion of $\Delta co2_{t-4}$, as might be suggested from the cross-correlation function, produced an insignificant coefficient estimate of $-0.20(\pm 0.13)$, while the restriction that the coefficients on $\Delta co2_t$ and $\Delta co2_{t-1}$ sum to zero was easily accepted. With this restriction imposed, the transfer function can be written as

$$z_t = 0.0148T_t - 0.327\Delta co2_t + \varepsilon_t,$$

$$\Delta\varepsilon_t = \left(1 - 0.442B - 0.284B^2\right)a_t, \qquad \hat{\sigma} = 0.1013.$$

Here $T_t = \Delta^{-1}I_t$ is a segmented trend such that $T_t = 0$ for $t \le 1970$ and $T_t = t - 1970$ for $t > 1970$. Apart from this segmented trend component, global temperatures are therefore a *negative* function of the current *growth* of $CO_2$ emissions, with this function being 'buried' in nonstationary noise.

This, perhaps surprising, negative relationship between temperatures and $CO_2$ emissions growth might be explained by arguing that transfer functions of this type are designed to model only the short-run interactions between these variables, when the relationship between temperatures and $CO_2$ emissions is essentially a long-run phenomenon. A natural way of examining if such a long run relationship exists is to consider whether the two series cointegrate. A convenient method of doing this in the above framework is to follow the approach of Pesaran et al. [12], which requires adding the lagged levels variables $z_{t-1}$ and $co2_{t-1}$ to the transfer function

$$\begin{aligned}\Delta z_t = {} & \underset{(\pm 0.114)}{0.0183I_t} - \underset{(\pm 0.142)}{0.320\Delta^2 co2_t} - \underset{(\pm 0.0186)}{0.0126z_{t-1}} - \underset{(\pm 0.0009)}{0.0002co2_{t-1}} \\ & + a_t - \underset{(\pm 0.081)}{0.438a_{t-1}} - \underset{(\pm 0.092)}{0.282a_{t-2}}, \qquad \hat{\sigma} = 0.1015.\end{aligned}$$

If these variables were significantly different from zero, then there would be a significant error correction term of the form $-0.0126(z_{t-1}+0.0159\, co2_{t-1})$, the expression in parentheses being the estimated cointegrating vector. It is clear, however, that these variables are completely insignificant and thus offer no evidence in favour of cointegration– indeed, the cointegrating vector would again imply that there was a negative long-run relationship between $CO_2$ emissions and temperatures (including a time trend as a further regressor does not alter this conclusion).[5] An alternative single equation approach, one that assumes cointegration, is that proposed originally by Phillips and Loretan [13] and Saikkonen [14], which leads to the following nonlinear models, depending on whether a linear or segmented trend is included

$$\begin{aligned} z_t &= \underset{(\pm 0.203)}{0.625} + \underset{(\pm 0.0017)}{0.0075}t - \underset{(\pm 0.093)}{0.468}co2_t + \underset{(\pm 0.164)}{0.319}\Delta co2_{t-1} + \underset{(\pm 0.065)}{0.602} \\ &\quad \times (z_{t-1} + 0.468co2_{t-1}) + a_t, \qquad \hat{\sigma} = 0.1013, \\ z_t &= -\underset{(\pm 0.067)}{0.280} + \underset{(\pm 0.0014)}{0.0060}T_t + \underset{(\pm 0.021)}{0.069}co2_t + \underset{(\pm 0.168)}{0.268}\Delta co2_{t-1} + \underset{(\pm 0.063)}{0.623} \\ &\quad \times (z_{t-1} - 0.069co2_{t-1}) + a_t, \qquad \hat{\sigma} = 0.1035. \end{aligned}$$

In both models, the significance of the lagged error correction term would seem to signify a cointegrating relationship, but with a linear time trend the negative long-run relationship between temperatures and $CO_2$ emissions continues to appear. Only with the inclusion of the segmented trend $T_t$ is a positive coefficient on $CO_2$ emissions obtained, but the fit of this model is markedly inferior to that of the linear trend specification.

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[5] Analysing the relationship between temperatures and $CO_2$ within a VAR/VECM framework uncovered marginal evidence in favour of cointegration (rejection of the null of no cointegration at approximately the 6% level) and an estimated cointegrating relationship that continued to give a negative, but even stronger, relationship between the two variables, being $z = 2.216 - 0.591co2 + 0.021t$.

## 6. Discussion

By following the empirical modelling principles laid down in Box and Jenkins [1], we have shown that annual global temperatures from 1850 may be represented by a drifting random walk with normally distributed and serially correlated innovations in the guise of an ARIMA$(0, 1, 2)$ process. Projections from this model out to 2100 produce a central increase of around 0.5ºC with a probability of less than one per cent of the lower bound of the IPCC 2100 projected increase in temperatures being reached. There are some indications, however, that the drift in the process may have shifted around 1970, as 0.975 forecast error bounds from this origin, calculated using parameter estimates obtained only from observations up to that year, are regularly breached by actual temperatures from 1998 onwards. Incorporating a segmented trend with a shift at 1970 allows the projected temperature increases to be substantially larger, with there now being more than a 60% probability of the IPCC lower bound increase being exceeded. (Clearly, there is still an infinitely small probability of the IPCC upper bound increase of 5.8ºC being reached.) The incorporation of such a segmented trend nevertheless begs the question of why such a shift in trend temperatures took place around 1970, for the ensuing warming trend could still be argued to be a consequence of natural variability in temperatures, which would be consistent with the results and perspectives of Cohn and Lins [3] and Mills [8, 9].

The incorporation of $CO_2$ emissions, whose logarithms follow a drifting random walk, as a driver of temperatures produces some intriguing results. Conventional transfer function modelling identifies a negative contemporaneous relationship between temperatures and the growth of $CO_2$ emissions buried in nonstationary noise of the ARIMA $(0, 1, 2)$ variety. This may be misleading as only a short-run relationship is capable of being modelled in this framework, but extensions to incorporate cointegration provide rather mixed results. The Pesaran et al. [12] approach finds no evidence of cointegration between temperatures and $CO_2$ emissions, but that of Phillips and Loretan [13] does find cointegration, although it requires the segmented trend to be included to yield a positive coefficient on emissions, and this at the expense of a deterioration in overall fit compared to a specification that includes just a linear trend.

This modelling exercise is both consistent with, and expands upon, the findings and views of Mills [8]. In essence, given the variety of alternative models of observed temperature records that have been considered recently (see, for example, the references contained in Mills [8]), there is no doubt that there are many ways, in the words of Kaufmann et al. [5], to 'skin the proverbial cat' of temperature modelling. Which of the alternatives should be chosen? Do you adopt a carefully specified univariate or transfer function model that, because of its property of adapting quickly to current movements in the series, essentially is unable to deliver much of an increase in forecasted temperatures; do you choose a simpler trend break model in which, the breaks are a consequence of presumably rare and large changes in key external forcing factors; or do you explicitly model the long-run, cointegrating relationship between temperatures and emissions or, more generally, radiative forcing, that is based on the hypothesis that changes in such variables, influenced in part by human activity, generate changes in temperatures. Statistical arguments alone are unlikely to settle issues such as these, but neither are appeals to only physical models or the output of computer simulations of coupled general circulation models. In such circumstances, it would appear that Mills' [8] quotation of another ageless proverb, 'you pays your money and you takes your choice' remains particularly apposite, given the ongoing debate concerning the potential costs of combating global warming and climate change.

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